Welcome, coman
The Quiz will expire (month-day-year) 10-01-2021, 01:00

Dr. Coman, PHY2054, Summary of equations, typical problems: Magnetism, Light, Nuclear, Quantum Mechanics

Section 1. Summary of equations:

The magnetic force exerted upon a charge q moving with velocity v in a magnetic field $\vec{B}$

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = q\vec{v} \cdot B \sin(\theta) \text{ where } \theta = \widehat{\left(\vec{v}, \vec{B}\right)}$$

Magnetic force on a straight conductor, length l, through which a current I circulates:

$$F = B \cdot I \cdot l \cdot \sin(\theta)$$

The force per unit length on each of two parallel wires separated by a distance d and carrying currents, I_1, I_2 and has the magnitude

$$\frac{F}{l} = \frac{\mu_o \cdot I_1 \cdot I_2}{2\pi \cdot d}$$

The forces are attractive if the currents are in the same direction and repulsive if they are in opposite directions.
The magnetic field at the center of a coil, length l, of N turns/loops of radius R, carrying a current I

$$B = \mu_o \cdot \frac{N}{l} \cdot I$$

Ampere's Law
The magnetic field at a distance r from a long, straight wire carrying current I

$$B = \frac{\mu_o \cdot I}{2\pi \cdot r}$$

$\mu_o = 4\pi \cdot 10^{-7} \frac{T \cdot m}{A}$

1.1 Magnetism

Theory: Electric charges in motion create/generate a cetor field, a magnetic field $\vec{B}$ Magnetic fields are produced by electric currents, which can be

1. macroscopic currents in wires,
2. microscopic currents associated with electrons in atomic orbits.
3.

The magnetic field $\vec{B}$ is defined in terms of the force exerted on moving charge $\rightarrow$ Lorentz force law. Since charges in motion generate magnetic fields, charges moving with a velocity $\vec{v}$ through an external magnetic field $\vec{B}$ would be subject to a Lorenz force:
$\vec{F}_{\text{Lorentz}} = q \cdot \vec{v} \times \vec{B}$
$F_{\text{Lorentz}} = q \cdot \vec{v} \cdot \vec{B} \cdot \sin(\theta)$
where θ is the angle between the vector velocity $\vec{v}$ and the vector magnetic field $\vec{B} \rightarrow$
$F_{\text{Lorentz}} = \text{maximum}$ when $\theta = 90^o \rightarrow \sin(90) = 1$
The magnetic/Lorentz force ,$\vec{F}_{\text{Lorentz}}$, is perpendicular onto both the $\vec{v}$ and $\vec{B}$ and its direction can be found out by rotating an imaginary right hand screw such that $\vec{v}$ overlaps $\vec{B}$; The direction of advancement of the right hand screw is the direction of $\vec{F}_{\text{Lorentz}}$
The magnitude of the magnetic field B:
$B = \frac{F_{\text{Lorentz}}}{q \cdot v \cdot \sin\theta} \rightarrow$ expressed in $\frac{N}{C \cdot \frac{m}{s}} = \text{Tesla}$

1.2 Lorentz Force sample problem:

A particle whose charge $q = +7.5\mu C$ and whose speed $v = 377 \frac{m}{s}$ enters a uniform magnetic field whose magnitude is $B = 0.50T$. Find the magnitude and direction of the magnetic force on the particle if the angle θ the velocity $\vec{v}$ makes with respect to the magnetic field $\vec{B}$ is 30^o.
Solution:
$F_{\text{Lorentz}} = q \cdot \vec{v} \times \vec{B}$
$F_{\text{Lorentz}} = 7.5\mu C \cdot 377 \frac{m}{s} \cdot 0.50T \cdot \sin(30^o) = 0.000707N = 707 \cdot 10^{-6}N$
The direction of the magnetic force exerted upon a positively charged particle is

given by the right hand rule : point fingers of right hand along $\vec{B}$ and thumb along $\vec{v}$; your palm points in the direction of $\vec{F}_{\text{Lorentz}}$ (on a positive charge).
If a negatively charged particle/electrons carrying the same charge would move with the same speed, in a $B = 0.5T$ the Lorentz Force would be:

$$F_{\text{Lorentz}} = -7.5\mu C \cdot 377\frac{m}{s} \cdot 0.50T \cdot \sin(30^o) = -0.000707N = -707 \cdot 10^{-6}N$$

The direction of the magnetic force exerted upon a negatively charged particle is given by the right hand rule : point fingers of right hand along $\vec{B}$ and thumb along $\vec{v}$;

The back of your palm points in the direction of $\vec{F}_{\text{Lorentz}}$ (on a negative charge).

1.3 Sample problem calculating the magnetic/Lorentz force exerted by a magnetic field $\vec{B}$ upon a charged particle

Calculate F_L - the magnitude - exerted by a uniform magnetic field, $B = 0.3 \cdot 10^{-4}T$ upon a charged particle, $q = -1.6 \cdot 10^{-9}C$ entering the region where the B is present with a velocity of $12\frac{m}{s}$ if the angle between the direction of the velocity and the magnetic field $\theta = 90^o$. $F_L = qvB\sin\theta$

$$\theta = 90^o \rightarrow \sin 90^o = 1$$

$$F_L = -1.6 \cdot 10^{-9}C \cdot 12\frac{m}{s} \cdot 0.3 \cdot 10^{-4}T = -5.76 \cdot 10^{-13}\text{Newtons}$$

1.4 Sample problem calculating the magnetic/Lorentz force exerted by a magnetic field $\vec{B}$ upon a charged particle

Calculate F_L - the magnitude - exerted by a uniform magnetic field, $B = 0.6G$ upon a charged particle, $q = 210 \cdot 10^{-9}C$ entering with a velocity of $320\frac{m}{s}$ into a region where a $\vec{B}$ is present, if the angle between the direction of the velocity and the magnetic field $\theta = 30^o$.

$$F_L = qvB\sin\theta$$

$$\theta = 30^o \rightarrow \sin 30^o = \frac{1}{1} = 0.5$$

$$F_L = ?$$

$$q = 210 \cdot 10^{-9}C$$

$$v = 320\frac{m}{s}$$

$$B = 0.6\text{Gauss}$$

$$B = 0.6\text{Gauss} \cdot 10^{-4}\frac{\text{Tesla}}{\text{Gauss}} \rightarrow \text{Tesla}$$

$$F_L = 210 \cdot 10^{-9} \cdot 3.20 \cdot 10^{2} \cdot 0.6 \cdot 10^{-4} \cdot \sin(30) = 2.0159 \cdot 10^{-9}$$

Section 2. Magnetic forces on Current carrying wires

2.1 Force exerted upon a wire carrying a current I

A long straight current carrying wire place in an external magnetic field $\vec{B}$ will experience a magnetic force;

The current I is the rate of flow of charges $\frac{q}{t}$;

The magnetic force is the sum of the Lorentz forces exerted upon each individual charge q_i:

$$F = \sum \left(q_i \cdot v \cdot B\right)$$

$$F = \sum q_i \cdot \left(\frac{L}{t}\right) \cdot B$$

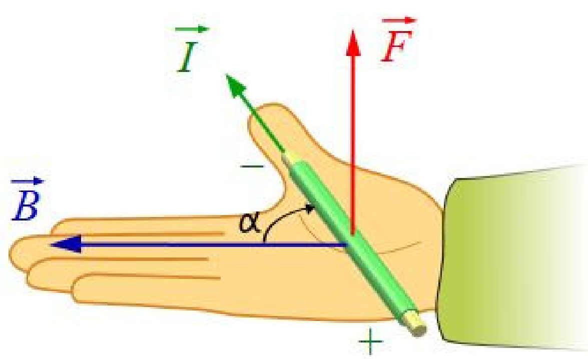
$$F = \sum \frac{q_i}{t} \cdot L \cdot B \rightarrow F = \frac{q_{\text{total}}}{t} \cdot L \cdot B = I \cdot L \cdot B$$

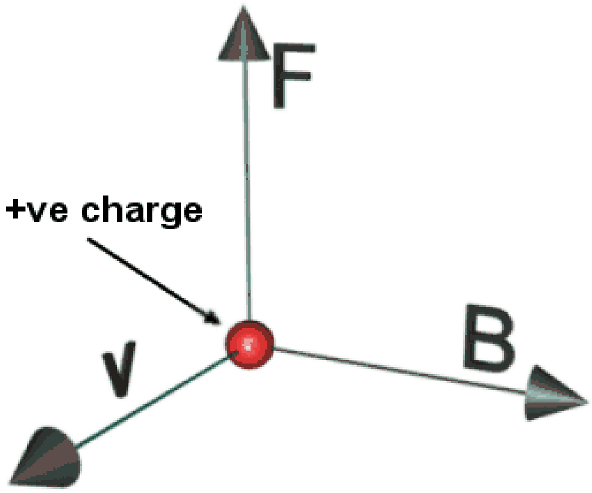
More generally the vector force is the cross product, $\times$, between the length of the wire as a vector and the vector magnetic , $\vec{B}$.

$$\vec{F} = I\,\vec{L} \times \vec{B} = I\,LB\ \sin\theta$$

The direction of the magnetic force on a straight wire carrying a current I is also given by the right habd rule.

When the fingers of the right hand point in the direction of the conventional current (dirrection in which a $+$ charge would move), curl your fingers towards the vector magnetic field $\vec{B}$, the extended thumb would point in the direction of the magnetic force on the wire.





2.2 Sample problem Magnetic Force exerted upon a current carrying wire

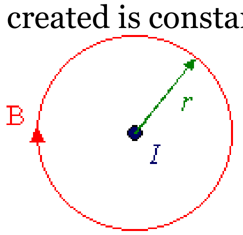
Estimate the maximum magnetic force that Earth's magnetic field could exert on a $5m$ long current-carrying wire in a $10A$ circuit in your house.
$B = 0.45 \times 10^{-4} T$.

- Solution:

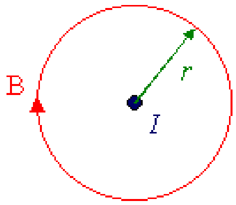
$\vec{F} = I\,\vec{L} \times \vec{B} = I\,LB\ \sin\theta$ when is the F maximum ? When $\theta = 90^{o}$.
$F = 10 \cdot 0.45 \cdot 10^{-4} \cdot 5 = 0.00225N$

Section 3. Ampere's Law

Ampere's law plays a role for magnetic fields that is similar to that played by Gauss's law for electric fields.
Ampere's law can be used to calculate the magnetic field in situations that are sufficiently symmetric. An important example is the magnetic field created by a long, straight wire through which a current I circulates. The magnetic field created is constant on any circular path around the wire.



The amount of current enclosed by this path is just I, the current flowing in the wire: The magnetic field from a current falls off as $\frac{1}{r}$, is inversely proportional with the distance. Recall that we saw a similar $\frac{1}{r}$ law for the electric field of a long line charge that also falls off as $\frac{1}{r}$.
This is not a coincidence. The direction of this field is in a circulational sense - the B field winds around the wire according to the right-hand rule.



Section 4. Charged Particles in Magnetic Fields

In a TV magnetic fields are used to direct a beam of electrons to different regions on the screen thus creating an image.

4.1 Applications: Charged Particles in Magnetic Fields

Charged particles can become trapped around magnetic field lines. Such trapping of solar wind particles has resulted in bands of charged particles around the Earth called Van Allen belts.

4.2 Calculating the radius of the circular trajectory of an electron trapped by the Earth's magnetic field

The magnetic force plays the role of a center seeking force, directed towards the center of the electron's circular trajectory whose radius is R.

$$F_{\text{Lorentz}} = qvB\sin(\theta) = 120 \cdot 10^{-3}C \cdot 12\frac{m}{s} \cdot 0.06T = 0.0864 \rightarrow N$$

$$F_{\text{Lorentz}} = F_{\text{centripetal}}$$

$$qvB\sin(\theta) = m_{\text{electron}}\frac{v^2}{R}$$

$$m_e = 511keV = 9 \cdot 10^{-31}kg$$
$$m_p = 1.67 \cdot 10^{-27}kg$$
$$B = 40{,}000 \cdot 10^{-9}T$$
$$\theta = 90^o \rightarrow \sin 90 = 1$$

$$R_{\text{electron}} = \frac{m_{\text{electron}}v}{q_{\text{electron}}B} = \frac{9 \cdot 10^{-31} \cdot 120}{-1.6 \cdot 10^{-19} \cdot 40000 \cdot 10^{-9}} \rightarrow \frac{10^{-31}}{10^4 \cdot 10^{-19} \cdot 10^{-9}} = 10^{-7} \rightarrow m$$

Similarly for a proton:

$$R_{\text{proton}} = \frac{m_{\text{proton}}v}{q_{\text{proton}}B}$$

$$|q_{\text{electron}}| = q_{\text{proton}} = 1.6 \cdot 10^{-19}C$$

Section 5. Magnetic flux, Theory, Definition

Magnetic flux is a measure of the magnetic field flowing through the unit surface area;
Magnetic flux : how much of the magnetic field flows through a specific area:
$\Phi_{\text{magnetic}}=\vec{B}\cdot\vec{A}B\cdot A\cdot\ \cos(\theta)$
where B represents the magnetic field and A represents the area of the surface.
The unit for magnetic flux is the Weber (Volt·seconds, or Tesla · meters^2).
For basic purposes, magnetic flux can be viewed as proportional to the number of field lines passing through the surface.
However, in reality it is the component of the field vector through a surface.
The magnetic flux is at its maximum when the vector for magnetic field and the area face the same direction (that is, when the angle between the two, θ, is zero), and is zero when the magnetic field vector is perpendicular to the area → similar to the maximum and minimum of the electric flux,

$\Phi_{\text{electric}}\ =\vec{E}\cdot\vec{A}=E\cdot A\cdot\ \cos(\theta)$.

5.1 Magnetic flux calculation, sample problem

A circular loop of wire of radius $r=0.5m$ is placed in a uniform magnetic field $B=2.0T$, with the plane of the loop perpendicular to the direction of the field, $\vec{B}$. Calculate the magnetic flux through the loop.
Solution:
Since $\vec{B}$ is perpendicular onto the plane of the loop → $\vec{B}$ is parallel to the unit vector perpendicular on the plane of the loop, $\hat{n}$. The magnetic flux is given by:
$\Phi_{\text{magnetic}}=B\cdot A\cdot\ \cos(\theta)=B\cdot\pi\cdot r^2\cdot\ \cos(\theta)$

$\Phi_{\text{magnetic}}\ =2T\cdot\pi\cdot(0.5m)^2\cdot\ \cos(0)=1.5707Wb$

5.2 Magnetic flux calculation, sample problem II

Find the magnetic flux through a 320 turn solenoid that has a length equal to $22.0cm$, has a radius equal to $2.00cm$, and carries a current of $2.00A$.
Formula:
The magnetic field inside a long solenoid:
$B=\mu_0\frac{N}{L}I$. where $\frac{N}{L}$ is the no of turns per unit length,

$\mu_o=1.26\cdot10^{-6}\frac{m\cdot kg}{s^2\cdot A^2}$.

$\Phi_{\text{mag}}=BA\cos\theta$
Solution:
$\Phi_{\text{mag}}\ =31.26\cdot10^{-6}\cdot320\cdot\frac{2}{0.22}\cdot2\cdot\pi\cdot\left(2\cdot10^{-2}\right)^2=0.000228\,T\cdot m^2$

$$\rightarrow \frac{m \cdot kg}{s^2 \cdot A^2} \cdot \frac{1}{m} \cdot A \cdot m^2$$

Section 6. Electromagnetic induction Induced emf: Faraday's Law and Lenz's Law

Whenever a magnet is moved in or out of a loop of wire a current is induced, electromotive force=voltage potential, emf:

$$\varepsilon = -\frac{\Delta\ \Phi}{\Delta\ t}$$

$$\varepsilon = -\frac{\Delta\ (B \cdot A \cdot\ \cos\ \theta)}{\Delta\ t}$$

The minus sign indicates the direction of the induced emf, which is given by Lenz's law.

6.1 Lenz's law → gives the direction of the induced electric current:

An induced emf in a wire loop or coil has a direction such that the current it creates produces its own magnetic field that opposes the change in magnetic flux through that loop or coil.

- Lenz's law is a consequence of the conservation of energy.

6.1 Induced emf Sample problem

A 500 turn rectangular loop of wire has an area per turn of $4.5 \cdot 10^{-3} m^2$ At $t = 0s$, a magnetic field is turned on, and its magnitude increases to $0.50T$ after $\Delta\ t = 0.75s$ have passed. The field is directed at an angle $\theta = 30^o$ with respect to the normal of the loop. (a) Find the magnitude of the average emf induced in the loop. (b) If the loop is a closed circuit whose resistance is $6.0\ \Omega$, determine the average induced current.

$$\varepsilon = -N \cdot \frac{\Delta\ \Phi}{\Delta\ t}$$

$$\varepsilon = -N \cdot \frac{\Delta\ (B \cdot A \cdot\ \cos\ \theta)}{\Delta\ t}$$

$$\text{(a) } |\varepsilon| = 500 \cdot \frac{0.5T \cdot 4.5 \cdot 10^{-3} m^2 \cdot\ \cos\ (30^o)}{0.75s} = 1.3V$$

$$\text{(b) } I = \frac{\varepsilon}{R} = \frac{1.3V}{6\ \Omega} = 0.22A$$

6.1 Transformers problem

Calculate the ratio of the voltage in the primary coil to the voltage in the

secondary coil for a step up transformer if the no of turns in the primary coil is $N_{\text{primary}} = 10$ and the no of turns in the secondary coil is $N_{\text{secondary}} = 10000$.
Solution:

$$\frac{N_{\text{primary}}}{N_{\text{secondary}}} = \frac{V_{\text{primary}}}{V_{\text{secondary}}} = \frac{10}{10000} = 10^{-3}$$

Section 7. Light's interaction wih matter

7.1 Refraction: Find the angle of refraction

A slab of glass that has an index of refraction of 1.50 is submerged in water that has an index of refraction of 1.33. Light in the water is incident on the glass. Find the angle of refraction if the angle of incidence is 60^0.
$n_{\text{water}} \sin\theta_{\text{water}} = n_{\text{glass}} \sin\theta_{\text{glass}}$

$1.33 \cdot \sin 60^o = 1.50 \cdot \sin\theta_{\text{glass}} \rightarrow$

$$\theta_{\text{glass}} = arc \sin\left[\frac{1.33 \cdot \sin 60^o}{1.50}\right] = arc \sin\left[\frac{1.33 \cdot 0.707}{1.50}\right] = 50.1567^o$$

7.2 Refraction: Determine the speed of light

Determine the speed of light in a material whose refractive index $n = 1.57$.

$$n = \frac{c}{v_{\text{material}}} \rightarrow$$

$$v_{\text{material}} = \frac{c}{n} = \frac{3 \cdot 10^8 \frac{m}{s}}{1.57} = 1.91 \cdot 10^8 \frac{m}{s}$$

7.3 Optics, lens' equation calculate distance image-lens

A candle is placed $14cm$ away from a concave lens.
The lens produces a virtual image $-8cm$ away from the lens.
If the object is moved back to $18cm$ from the lens, determine the position of the image with respect to the lens?

Hint: $\frac{1}{f} = \frac{1}{O} + \frac{1}{I}$
Use this interactive figure to verify your calculations! Click on the Understand tab at the right bottom to change the distances $O_{\text{object lens}} = p$ and $I_{\text{image-lens}} = q$!
Solution:

$$\frac{1}{f} = \frac{1}{O} + \frac{1}{I}$$

$O = -14cm$ since the axis of symmetry of the lens is the origin → all distance to the left of the lens are negative;
Virtual image → is formed on the same side of the lens as the object; →
$I = -8cm$.

$$\text{focal length} = f = \frac{1}{\frac{1}{-14} + \frac{1}{-8}} = -5.1cm$$

$$O = \frac{1}{\frac{1}{(-5.1) - \frac{1}{18}}} = -5.15cm$$

Section 8. Nuclear reactions

8.1 Calculate the no of radioactive nuclei left after a time t has passed.

Calculate the no of radioactive nuclei left after a time $t = 178\ \text{years}$ has passed.
Known: intial no of radioactive/unstable nuclei $N_o = 100$; Nucleus is Cs^{137} whose half life $t_{\frac{1}{2}} = 30\ \text{years}$

Decay law:

$$N_{\text{after time t has passed}} = N_0 \cdot e^{-\lambda \cdot t}$$

$$N_{\text{after time t has passed}} = N_0 \cdot exp(-\lambda \cdot t)$$

$$\lambda \cdot t_{\frac{1}{2}} = 0.693 = \ln(2)$$

$$N_{\text{after time t has passed}} = 100 \cdot e^{-\left(\frac{0.693}{t_{\frac{1}{2}}}\right) \cdot t}$$

$$N_{\text{after time t has passed}} = 100 \cdot e^{-\frac{0.693}{30} \cdot 178} = 1.63782$$

8.2 Calculate the no of radioactive nuclei left after a time t has passed.

Calculate the no of radioactive nuclei left after a time $t = 30\ \text{years}$ has passed.
Known: intial no of radioactive/unstable nuclei $N_o = 100$; Nucleus is Cs^{137} whose half life $t_{\frac{1}{2}} = 30\ \text{years}$

Decay law: $N_{\text{after time t has passed}} = 100 \cdot e^{-\frac{0.693}{30} \cdot 30} = 50.007$

8.3 Nuclear reactions: calculating time after which initial no. of nuclei decreases by a factor $n = 1000$

If the half life of Cs^{137} is 30 years how much time will be required to reduce a 1 kg sample to 1g ?

$$\ln\left(\frac{N}{N_0}\right)=-\lambda\cdot t\rightarrow$$

$$t=\frac{\ln\left(\frac{N}{N_0}\right)}{-\lambda}=\frac{\ln\left(\frac{1}{1000}\right)}{\frac{0.693}{30}}\rightarrow 299 years$$

8.4 Nuclear reactions: decay constant

Determine the $t_{\frac{1}{2}}$ of U^{238} if the decay constant is $4.87\cdot 10^{-18}\frac{1}{s}$ $t_{\frac{1}{2}}\cdot\lambda=0.693\rightarrow$

$$t_{\frac{1}{2}}=\frac{0.693}{\lambda}=\frac{0.693}{4.87\cdot 10^{-18}\frac{1}{s}}\rightarrow seconds$$

8.5 Nuclear reactions: calculating time after which initial no. of nuclei decreases by a factor $n=1000$

If the half life of Cs^{137} is 30 years how much time will be required to reduce a 1 kg sample to 1g ?

$$\ln\left(\frac{N}{N_0}\right)=-\lambda\cdot t\rightarrow$$

$$t=\frac{\ln\left(\frac{N}{N_0}\right)}{-\lambda}=\frac{\ln\left(\frac{1}{1000}\right)}{\frac{0.693}{30}}\rightarrow 299 years$$

8.6 Radioactive decay, Carbon dating: calculating how long ago a sample of a once living organism- was alive

How long ago an organism was alive if in a sample of that once living organism the no of C^{14} radioactive nuclei is $N_{f=}2400$ and the no of C^{14} radioactive nuclei in a similar living sample is $N_0=12800$. Assume that the only way of acquiring C^{14} was through ingestion and in the time that has passed no other prcesses were responsible for producing radioactive C^{14} but neutron capture by N^{14}.
Solution: The equation governing radioactive decay
$N(t)=N_{\text{initial}}\cdot e^{-\lambda\cdot t}\rightarrow$

$$\frac{N_{\text{final}}}{N_{\text{initial}}}=e^{-\lambda \cdot t}$$

$$\frac{2400}{12800} = e^{-\lambda \cdot t}$$

$$\lambda_{C^{14}} = \frac{\ln(2)}{t_{\frac{1}{2}}} = \frac{0.693}{5730\ \text{years}} \rightarrow$$

$$\ln\left(\frac{2400}{12800}\right) = \ln\left(e^{-\lambda \cdot t}\right)$$

$$\ln\left(\frac{2400}{12800}\right) = -\lambda \cdot t \cdot \ln(e) \rightarrow$$

$$\frac{\ln\left(\frac{2400}{12800}\right)}{-\lambda} = t$$

$$t = \frac{\ln\left(\frac{2400}{12800}\right)}{-\frac{0.693}{5730\ \text{years}}} = 13841.10\text{years}$$

Section 9. Quantum mechanics

9.1 The De Broglie wavelength

Determine the wavelength associated with a baseball ball whose mass $m = 0.86kg$ and speed $v = 44\frac{m}{s}$.

Solution: $h = \lambda_{\text{baseball ball}} \cdot p = \lambda_{\text{baseball ball}} \cdot (m \cdot v)$

$$h = 6.626 \cdot 10^{-34} J \cdot s$$

$$\lambda_{\text{baseball}} = \frac{h}{m \cdot v} = \frac{6.626 \cdot 10^{-34} J \cdot s}{0.86 \cdot 44} kg \cdot \frac{m}{s} = 1.75 \cdot 10^{-35} m$$

Section 10. Relativity NOT REQUIRED FOR SUMMER 2021

10.1 Calculating the γ factor

Calculate the γ factor of a car that is moving at $80\,\%$ of the speed of light, c. Knowing $c = 3 \cdot 10^8 \frac{m}{s}$.

$$\gamma = \sqrt{\frac{1}{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\gamma = ? = \frac{1}{\sqrt{1-\left(\frac{0.80\cdot 3\cdot 10^8\frac{m}{s}}{3\cdot 10^8\frac{m}{s}}\right)^2}} = 1.67 \rightarrow \text{unitless}$$

10.2 Calculating the length contraction using the above γ factor

How long would a $2.5m$ car look like as measured by an observer moving at $80\,\%$ of $c = 3\cdot 10^8\frac{m}{s}$?

$$L = \frac{L_{\text{rest}}}{\gamma}$$

$$L_{\text{rest}} = 2.5m \rightarrow$$

$$L = \frac{L_{\text{rest}}}{\gamma} = \frac{2.5m}{1.67} = 1.49m$$

10.3 Relativity important: length contraction

Calculate the length of a car moving at $0.6c$ where c is the speed of light; The length of the car at rest is $2.5m$.

$$L_{\text{car in motion at 0.6 of c}} = ? = \frac{L_{\text{rest}}}{\gamma}$$

$$L_{\text{car in motion at 0.6 of c}} = ? = \frac{L_{\text{rest}}}{\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}} = L_{\text{rest}}\cdot\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$L_{\text{rest}} = 2.5m$$

$$\gamma = \frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}} = \frac{1}{\left[1-\left(\frac{0.6\cdot c}{c}\right)^2\right]^{\frac{1}{2}}} = 1.25$$

$$L_{\text{car in motion at 0.6 of c}} = ? = \frac{L_{\text{rest}}}{\gamma} = \frac{2.5m}{1.25} = 2.5\cdot 0.8 = 2m$$

10.4 Energy mass equivalence

Calculate $E_{\text{of an apple}}$ whose mass is $m = 50g$.

$E = m \cdot c^2$

$$E_{\text{from an apple}} = \frac{50}{1000} kg \cdot \left(3 \cdot 10^8\right)^2 = 5 \cdot 10^{-2} \cdot 9 \cdot 10^{16} \rightarrow \text{Joules}$$

Assuming we need 2500 kilocalories per day to function $\rightarrow =$

$$2,500,000 \text{calories} \cdot 4.454 \frac{\text{Joules}}{\text{calore}} = 11,135,000 \text{Joules}$$

$$E_{\text{needed for a 70 years lifetime}} = 11135000 \text{Joules} \cdot \left(70 \text{years} \cdot 365 \frac{\text{days}}{\text{year}}\right) = 2.8 \cdot 10^{12} J$$

TASK: calculate the ratio: $\frac{E_{\text{from an apple}}}{E_{\text{needed for the entire life}}} = ?$

$$\frac{E_{\text{from an apple}}}{E_{\text{needed for the entire life}}} = \frac{5 \cdot 10^{-2} \cdot 9 \cdot 10^{16}}{2.8 \cdot 10^{12}} = 1607 \rightarrow$$

$$E_{\text{from an apple}} > E_{\text{needed for the entire life}} !!$$