PHY2054, Summary of equations

0.1 Celsius Fahrenheit conversion

$$T_F = 1.8 \cdot T_C + 32$$

0.1 Fahrenheit Celsius conversion

$$T_C = \frac{T_F - 32}{1.8}$$

0.1 Celsius Kelvin conversion

$$T_K = T_C + 273.15$$

0.1 Thermal Expansion

All objects change size with changes in temperature.
For a temperature change ΔT, a change ΔL in any linear dimension L:
$$\Delta L = L_{\text{initial}} \cdot \alpha \cdot (T_{\text{final}} - T_{\text{initial}})$$

$\alpha =$ coefficient of linear expansion.

The change ΔV in the volume V of a solid or liquid:

$$\Delta V = V_{\text{initial}} \cdot \beta \cdot (T_{\text{final}} - T_{\text{initial}})$$

$\beta =$ coefficient of volumetric expansion.

0.1 Heat Q

Energy that is transferred in or out of a thermodinamic system:
$Q = C \cdot (T_{\text{final}} - T_{\text{initial}}) \rightarrow \text{Joules or calories (cal)}$
between a system and its environment because of a temperature difference between them.
It can be measured in joules (J), calories (cal), kilocalories (Cal or kcal), or British thermal units (Btu), with
$C =$ Heat Capacity

For an object whose mass $= m$ the heat absorbed or given off -up to a sign- is:

$$Q = m \cdot c \cdot (T_{\text{final}} - T_{\text{initial}})$$

where c is the specific heat $\rightarrow \frac{\text{cal}}{\text{kg} \cdot ^o C}$ or $\frac{\text{Joule}}{\text{kg} \cdot ^o C}$

0.1 Heat absorbed during a phase change/transition

During a phase change solid→liquid, liquid→gas, liquid→solid the temperature, T is constant.

$$Q_{\text{absorbed or given off during a phase change}} = m \cdot L$$

$L =$ latent heat, a characteristic of a material, is the amount of energy as heat per unit mass,
required to change the state (but not the temperature) of a particular material,
sometimes called heat of transformation.

$$Q_{\text{absorbed so water}\rightarrow\text{vapor}} = m \cdot L_{\text{vaporization}}$$

0.1 Work Associated with Volume Change

A gas may exchange energy with its surroundings through work.
The work done by a gas, W as it expands or contracts
from an initial volume V_{initial} to a final volume V_{final}:

$$W = (nRT)\ \ln\left(\frac{V_{\text{final}}}{V_{\text{initial}}}\right) \text{ for } T = \text{constant} - \text{isothermal process}$$

0.1 Graphical interpretation of work done by a gas/thermodynamic system:

On a pressure-along y, p versus volume, V graph, the work done is the are underneath the curve.

0.1 First Law of Thermodynamics

The principle of conservation of energy for a thermodynamic process

$$\Delta\ E_{\text{internal}} = E_{\text{internal, final}} - E_{\text{internal, initial}} = Q - W$$

$$E_{\text{internal}} = Q_{\text{transferred in}} - W_{\text{done by gas}}$$

0.1 Summary of Thermodynamic Changes; Formulas

$$C_p = \text{heat capacity at constant pressure} = \frac{Q_{\text{pressure=constant}}}{n \cdot\ \Delta\ T}$$

$$C_V = \text{heat capacity at constant volume} = \frac{Q_{\text{volume=constant}}}{n \cdot\ \Delta\ T}$$

$\Delta\ S =$ change in entropy
$\Delta\ U = \ \Delta\ E_{\text{internal}} =$ change in internal energy of a thermodynamic system

0.1 Specific Processes

isothermal $\rightarrow\ \Delta\ T = 0 \rightarrow T = \text{constant}$
isochoric $\rightarrow\ \Delta\ V = 0 \rightarrow V = \text{constant}$
isobarric $\rightarrow\ \Delta\ p = 0 \rightarrow p = \text{constant}$
adiabatic $\rightarrow Q = 0 \rightarrow$ no heat is transferred in or out of the thermodynamic system

$n =$ number of moles

$R =$ universal gas constant $= 8.3145 \frac{J}{\text{mol} \cdot K}$

$N =$ number of molecules

$k =$ Boltzmann constant $= 1.38066 \cdot 10^{-23} \frac{J}{K} = 8.617385 \cdot 10^{-5} \frac{eV}{K}$

$$k = \frac{R}{N_A}$$

$N_A =$ Avogadro's number $= 6.0221 \cdot 10^{23} \frac{\text{particles}}{\text{mol}}$

Molar specific heat at constant pressure:

$$Q = nC_p \, \Delta T$$

$$C_p = C_V + nR$$

For an ideal monoatomic gas :

$$C_p = \frac{5}{2} R = 20.8 \frac{J}{\text{mol} \cdot K}$$

$$C_V = \frac{3}{2} R = 12.5 \frac{J}{\text{mol} \cdot K}$$

0.1 Molar Heat Capacities for different Gases

Process	Equation	Work done W (Joules)	Heat absorbed/given off Q (Joules)
Isobaric	$p =$ constant$\rightarrow \Delta p = 0$ $pV = nRT \rightarrow$ $\frac{V}{T} = \text{constant} = \frac{nR}{p}$	$W = -p \, \Delta V$	$Q = C_p \cdot \Delta T$
Isochoric	$V =$ constant$\rightarrow \Delta V = 0$ $pV = nRT \rightarrow$ $\frac{p}{T} = \text{constant} = \frac{nR}{V}$	$W = p \, \Delta V = p \cdot 0 = 0$	$Q = C_V \cdot \Delta T$
Isothermal	$T =$ constant$\rightarrow \Delta T = 0$ $pV = nRT$ rarr $pV = \text{constant} = (nRT)$	$W = Q = nrT \cdot \ln\left(\frac{V_{\text{final}}}{V_{\text{initial}}}\right)$	$Q = W = nrT \cdot \ln\left(\frac{V_{\text{final}}}{V_{\text{initial}}}\right)$
Adiabatic	$Q = 0$ $pV^{\gamma} =$ constant	$W = \frac{p_{\text{final}} V_{\text{final}} - p_{\text{initial}} V_{\text{initial}}}{\gamma - 1}$	$Q = 0 \rightarrow$ no heat absorbed or given off
isoentropic	$S =$ entropy $=$ constant $\rightarrow$ $\Delta S = 0$ $TV^{\gamma - 1} =$ constant	$W = C_V \cdot \frac{p_{\text{final}} V_{\text{final}} - p_{\text{initial}} V_{\text{initial}}}{nR}$	

Process	Equation	Change in internal Energy of the gas ΔU or ΔE_{int} (Joules)	Change in entropy ΔS (Joules)
Isobaric	$p = \text{constant} \rightarrow \Delta p = 0$ $pV = nRT \rightarrow$ $\frac{V}{T} = \text{constant} = \frac{nR}{p}$	$\Delta U = C_V \cdot \Delta T$	$\Delta S = C_p \cdot \ln\left(\frac{V_{\text{final}}}{V_{\text{initial}}}\right)$ $\Delta S = C_p \cdot \ln\left(\frac{T_{\text{final}}}{T_{\text{initial}}}\right)$
Isochoric	$V = \text{constant} \rightarrow \Delta V = 0$ $pV = nRT \rightarrow$ $\frac{p}{T} = \text{constant} = \frac{nR}{V}$	$\Delta U = C_V \cdot \Delta T$	$\Delta S = C_V \cdot \ln\left(\frac{p_{\text{final}}}{p_{\text{initial}}}\right)$ $\Delta S = C_V \cdot \ln\left(\frac{T_{\text{final}}}{T_{\text{initial}}}\right)$
Isothermal	$T = \text{constant} \rightarrow \Delta T = 0$ $pV = nRT$ rarr $pV = \text{constant} = (nRT)$	$\Delta E_{\text{internal}} = 0$	$\Delta S = nR \cdot \ln\left(\frac{V_{\text{final}}}{V_{\text{initial}}}\right)$ $\Delta S = nR \cdot \ln\left(\frac{p_{\text{final}}}{p_{\text{initial}}}\right)$
Adiabatic	$Q = 0$ $pV^{\gamma} = \text{constant}$	$\Delta U = \frac{p_{\text{final}}V_{\text{final}} - p_{\text{initial}}V_{\text{initial}}}{\gamma - 1}$	0
isoentropic	$S = \text{entropy} = \text{constant}$ $\rightarrow$ $\Delta S = 0$ $TV^{\gamma-1} = \text{constant}$	$\Delta U = C_V \cdot \frac{p_{\text{final}}V_{\text{final}} - p_{\text{initial}}V_{\text{initial}}}{nR}$	$\Delta S = 0$

0.1 Ideal Gas Law

0.1 The Ideal Gas Law problems

$$pV = nRT$$

$$pV = Nk_{\text{Boltzmann}} \cdot T$$

$$p = pressure, \quad V = \text{volume}, \quad T = \text{temperature},$$

$$n = \text{no of moles}, \quad N = \text{no of particles/molecules}.$$

$$R = 8.31 \frac{J}{mol \cdot {}^{o} K} = \text{universal/ideal gas constant}$$

$$R = 0.0082 \frac{\text{Liters} \cdot \text{atm}}{K \cdot \text{mol}}$$

$$k_B = \text{Boltzmann's constant} = 1.38 \cdot 10^{-23} \frac{J}{K}$$

When using the ideal gas law make sure the temperature is in Kelvin.

The pressure can be used in atm if the V is in mL or in Liters.

If pressure is in $\frac{N}{m^2}$ use V in m^3 → work done will be in Joules!→

$$W = p \cdot \Delta V \rightarrow \frac{N}{m^2} \cdot m^3 = N \cdot m = \text{Joule}$$

0.1 The first principle of thermodynamics

→ 1^{st}principle
an energy conservation principle applied to thermodynamic systems
$\Delta U_{\text{internal}} = \Delta E_{\text{internal}} = Q_{in} + W_{\text{on}}$
Internal energy is a function of temperature only, does not depend on any other physical property, just T:

$$U_{\text{internal}} = E_{\text{internal}} = f(T) = \frac{3}{2} k_B T$$

$$\frac{3}{2} k_B dT = \delta Q_{in} - pdV$$

$\Delta T = 0$ isothermal → $\Delta E_{\text{internal}} = 0$

$Q_{in} = -W_{\text{onto the system}} = W_{\text{by the system}}$

$\Delta V = 0$ → isochoric → $W = 0$

$\Delta U_{\text{internal}} = Q_{in}$

$\Delta Q = 0$ → adiabatic →

$\Delta U_{\text{internal}} = W_{on}$

0.1 1st principle of thermodynamics, sample problem

1. $9000J$ of heat, Q_{in} is added to a gas;
The gas expands and does $W_{\text{by gas}} = 2000J$ of work.
Calculate the change in the internal energy of the gas, E_{internal} ? $Q_{\text{in}}=9000J$
$W_{\text{by gas}} = 2000J$
$\Delta E = Q_{\text{in}} + W_{\text{on the system}}$ →
$\Delta E = Q_{\text{in}} - W_{\text{by the system}}$ →
$\Delta E = 9000J - 2000J = 7000J$

0.1 1st principle of thermodynamics, sample problem

One mole of an ideal diatomic gas is heated at constant volume from $T_{\text{initial}} = 300$ to $T_{\text{final}} = 500K$.
A) Calculate the increase in internal energy, the work done and the heat added.
B) Calculate the increase in internal energy, the work done and the heat added as $T_{\text{initial}} = 300$

increases to $T_{\text{final}} = 500K$ but the pressure is kept constant.

A) Solution:

(a) The increase in internal energy $\Delta U = C_V \cdot \Delta T$

For a diatomic gas

$C_V = \frac{5}{2}R$

$\Delta T = 500 - 300 = 200K \rightarrow$

$\Delta U = \frac{5}{2}RdT \rightarrow$

$dU = \frac{5}{2} \cdot 8.315 \cdot 200 = 4157.5J$

since $W = p \cdot \Delta V$ since $\Delta V = 0 \leftarrow$ isochoric$\rightarrow W = 0$

From the 1st principle of thermodynamics:

$Q = \Delta U + W \rightarrow$

$Q = \Delta U = 4157.5J$

A) Solution: Internal energy is a function of temperature only:

$U = f(T)$

$dU = \frac{5}{2} \cdot 8.315 \cdot 200 = 4157.5J$

$W = p \cdot \Delta V$

$p_1 V_1 = nRT_1$

$p_2 V_2 = nRT_2$

Subtracting one equation from another $\rightarrow p(V_2 - V_1) = R(T_2 - T_1) \rightarrow$

$W = p(V_2 - V_1 = R(T_2 - T_1)$

0.1 1st principle of thermodynamics, sample problem

2. 2 moles of an ideal monatomic gas undergoes the following process:

It starts in the state ($p_o = 1atm, V_o = 2L$);

Then it expands isobarically to the state ($p_o, 5V_o$);

Then it is heated at constant volume (isochorically) to ($7p_o, 5V_o$). $T_o = 24^oC$.

a) Plot this on a PV diagram

b) What is the temperature difference between the initial and the final state?

c) What is the internal energy change?

d) What is the total heat flow into the gas?

$\frac{p_o V_o}{T_o} = \frac{7p_o 5V_o}{T_f} \rightarrow$

$35 \cdot T_o = T_f \rightarrow$

$\Delta T = T_{f\,-} T_o = 35T_o - T_o = 34T_o = 34 \cdot (24 + 273.15) \rightarrow T_f$ in Kelvin

c) $\Delta E = \frac{3}{2} k_B \cdot \Delta T$

d) $\Delta E = Q_{in} + W_{on} = Q_{in} - W_{by}$

$$Q_{in} = \Delta E + W_{by} = \frac{3}{2} k_B \cdot \Delta T + p_o \Delta V$$

$$Q_{in} = = \frac{3}{2} k_B \cdot \Delta T + p_o (5V_o - V_o)$$

$$Q_{in} = = \frac{3}{2} k_B \cdot 34 \cdot (24 + 273.15) + 1(5 \cdot 2 - 2)$$

0.1 Thermal expansion

The linear -1 dimensional- thermal expansion coefficient α:

$$\alpha = \frac{L_{f\_} L_i}{L_i} \cdot \frac{1}{\Delta T}$$

0.1 Thermal expansion sample problem

What is the change in length of a column of mercury $3cm$ long
if its T increases from 37^oC to 41^oC?

The linear expansion coefficient of Hg is $60 \cdot 10^{-6} \frac{1}{.^o C}$.

$$\Delta L = (L_{f\_} L_i) = L_i \cdot \alpha \cdot \Delta T = 3cm \cdot 60 \cdot 10^{-6} \frac{1}{.^o C} \cdot (41^oC - 37^oC)$$

0.1 Thermal processes: the variation of electrical resistance with temperature

A thermistor is a solid-state device widely used in a variety of engineering applications. Its primary characteristic is that its electrical resistance varies greatly with temperature. Its temperature dependence is given approximately by

$$R = R_0 e^{\frac{B}{T}}$$

where R is in ohms Ω, T is in kelvins, and R_0 and B are constants that can be determined
by measuring R at calibration points such as the ice point and the steam point.
(a) If $R = 7360\ \Omega$ at the ice point and $153\ \Omega$ at the steam point, find R_0 and B.

$$7360\ \Omega = R_0 e^{\frac{B}{273K}}$$

$$153\ \Omega = R_0 e^{\frac{B}{373K}}$$

dividing eq (1) by eq ,(2) $\rightarrow B \rightarrow$ Plug B into eq (1) $\rightarrow R_0$.

$$R(T = 0^oC = 273^oK) = 7360\ \Omega = R_0 \cdot e^{\frac{B}{273}}$$

$$R(T=0^oC=273^oK)=7360\ \Omega\ =R_0\cdot e^{\frac{3944.12}{273}}$$

$$R_0=\frac{7360}{e^{\frac{3944.12}{273}}}=0.0039$$

$$R(T=100^oC=373^oK)=153\ \Omega\ =R_0\cdot e^{\frac{B}{373}}$$

$$\ln(48.10)=\ \ln\left[e^{\frac{B}{273}-\frac{B}{373}}\right]=\left(\frac{B}{273}-\frac{B}{373}\right)\cdot\ \ln e=\frac{373B-273B}{273\cdot 373}$$

$$B=\frac{273\cdot 373\cdot\ \ln 48.10}{100}=?=3944.124^oK$$

$$\rightarrow B=?\ \frac{7360}{153}\left(\frac{\Omega}{\Omega}\right)=\frac{R_0}{R_0}\cdot\left(e^{\frac{B}{273}-\frac{B}{373}}\right)$$

$$R(T)=R_0\cdot e^{\frac{B}{T}}$$
$$B\rightarrow^o K$$
$$R_0\rightarrow\ \Omega$$
$$R(T=98.6^oF\rightarrow ?^oC\rightarrow ?^oK)=R_0\cdot e^{\frac{B}{T}}=0.0039\cdot\left(e^{\frac{3944.12}{\left(\frac{98.6-32}{1.8}\right)+273}}\right)=1307^oK$$

$$R_o=?$$

b) Converting 98.6^oF to kelvins $\rightarrow$

$$R(T)=R_0 e^{\frac{B}{310K}}$$

0.1 ENERGY CONVERSION sample problem: Potential gravitational Energy into Heat:

Determine the T change for a lead ball dropped from 20 meters assuming that only half of the thermal energy generated on impact goes into the ball.

$$m\cdot g\cdot\frac{h}{2}=Q=m\cdot s\cdot\Delta\ T\rightarrow$$

$$\Delta\ T=\frac{g\cdot h}{2\cdot s}=\frac{g\cdot 10}{2\cdot s}=\frac{9.8\,\frac{m^2}{s^2}\cdot 10m}{2\cdot 0.03\,\frac{kcal}{kg\cdot{}^oC}\cdot 4186\,\frac{\left(kg\cdot\frac{m^2}{s^2}\right)}{kcal}}\rightarrow$$

$$\Delta\ T=\frac{98}{2\cdot 0.03\cdot 4186}=0.07^oC$$

0.1 Heat transferred:

How much energy does a refrigerator remove from $100.0g$ of water at 20.0^oC to make ice at (minus) -10.0^oC?

Three steps:

1. Liquid water at$20.0^oC \rightarrow$ liquid water at0^oC

$Q_1{=}Q_{20.0^oC\rightarrow 0^oC}{=}m\cdot c_{\text{water}}\cdot\ \Delta T$ to cool from 20.0^oC to$0^oC \rightarrow$

$$Q_1{=}100.0g{\cdot}1.00\frac{cal}{g^oC}{\cdot}(0.0^oC-20.0^oC){=}{-}2,000cal$$

2. Liquid water at$0.0^oC \rightarrow$ solid ice at0^oC : $L\rightarrow S$ phase change

$Q_2{=}Q_{L=0.0^oC\rightarrow S=0^oC}{=}m\cdot L_{\text{fusion}}$

$$Q_2 = 100.0g{\cdot}334\frac{J}{g} = 33,400J = 7979.358\ \text{ cal}$$

but since we know this is supplied heat $\rightarrow$ absorbed by water $\rightarrow$ $Q_2 = -7979.358\ \text{ cal}$

3. Solid ice at$0.0^oC \rightarrow$ solid ice at -10^oC

$Q_3{=}Q_{iceat0.0^oC\rightarrow iceat-10^oC}{=}m\cdot c_{\text{ice}}\cdot\ \Delta T$

$$Q_3 = 100.0g{\cdot}0.5\frac{cal}{g^oC}{\cdot}(-10.0^oC-0.0^oC) = -500\text{cal}\rightarrow$$

$$Q_{\text{total supplied}} = Q_1+Q_2+Q_3 = -2000\text{cal}-7979.358\ \text{ cal}\ -500\text{cal} = 10479.358\text{cal}$$

0.1 Specific Heat

Specific heat c is the proportionality constant between heat transferred and change in temperature, $\Delta T = T_{f\_}T_i$:
$Q = m\cdot c\cdot\ \Delta T{=}C\cdot\ \Delta T$
where C (capital **C** is the heat capacity:$C = m\cdot c$.

The specific heat of aluminum is more than twice the specific heat of copper. A block of copper and a block of aluminum have the same mass and temperature (20^oC). The blocks are simultaneously dropped into a single calorimeter containing water at 40^oC.
Which statement is true when thermal equilibrium is reached?
(a) The aluminum block is at a higher temperature than the copper block.
(b) The aluminum block has absorbed less energy than the copper block.
(c) The aluminum block has absorbed more energy than the copper block.
(d) Both (a) and (c) are correct statements.

0.1 Entropy: defined for irreversible processes:

$$\Delta S_{\text{irreversible}} = \frac{Q}{T}$$

$$S = \text{entropy} = k_B \cdot \ln M$$

$$\Delta E_{\text{internal}} = Q_{in} + W_{on} = \Delta S \cdot T + p \Delta V$$

$$S = \text{entropy} = \text{how much energy is unavailable to do work}$$

The 1^{st} principle of thermodynamics applied to the free expansion of a gas $\rightarrow$ expansion in vacuum:
A gas trapped in half a tube;

The other half separated by a wall from the 1st half, the second half is empty $\rightarrow$ vacuum

$$W = 0$$

$$\Delta E = Q - W_{by} = 0 \rightarrow$$

$$E_{\text{internal}} = f(T) = 0 \rightarrow \Delta T = 0$$

$$pV = nRT$$

$$p_1 \cdot V_1 = p_2 \cdot 2V_1$$

$$p_2 = \frac{p_1}{2}$$

$$V_2 = 2V_1$$

0.1 Efficiency of thermal engines

Modern automobile gasoline engines have efficiencies of about $25\,\%$. About what percentage of the heat of combustion is not used for work but released as heat?

(a) $25\,\%$,
(b) $50\,\%$,
(c) $75\,\%$,
(d) $100\,\%$,
(e) You cannot tell from the data given.

If a heat engine does $100kJ$ of work per cycle while releasing $400kJ$ of heat, what is its efficiency?

(a) $20\,\%$,
(b) $25\,\%$,
(c) $80\,\%$,
(d) $400\,\%$,